ON THE TWO-COMPONENT SUNSPOT MODEL

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It is shown that the taking of inhomogeneities into consideration involves great changes in sunspot models. Several dense inhomogeneities may cause an increase of the spot opacity up to values comparable with the opacity of the photosphere. Taking of the inhomogeneities into account would allow a rarefied but optically thick model of the sunspot to be built.

Двухкомпонентная модель солнечного пятна. Показано, что учёт неоднородностей приводит к сильным изменениям в моделях солнечных пятен. Небольшое количество плотных неоднородностей приводит к сильному падению прозрачности пятна вплоть до величин, сравнимых с прозрачностью фотосферы. Учёт неоднородностей позволяет построить разреженную, но оптически плотную модель солнечного пятна.

1. *Introduction*

Available models of solar spots are divided into two types of models: rarefied and non-rarefied ones. Models of the first type (Michard, 1953; Fricke and Elsässer, 1966, Van't Veer, 1962) as a rule use a minimum of a priori assumptions. The gas pressure of these models is much smaller than that of the photosphere at corresponding optical depths. The optical depth $\tau^* \sim$ I is reached at very great geometric depths, $h \sim 2000-3000$ km. This causes very strong deviations from hydrostatic equilibrium. Thus, in the Fricke and Elsässer (1965) model an effective gravity acceleration is 10 times less than that in the photosphere. Such a great depth is in disagreement with the sharp boundaries of the umbra and penumbra. Moreover, as Jensen and Maltby (1965) showed, rarefied models near the solar limb must cause quite an unreal brightness in the spot: the spot should become a much brighter formation than the photosphere.

Models of the second type (Mattig, 1958; Jakimiec, 1964, 1965; Zwaan, 1965) lead to spot pressure values near to the photospheric ones. Either a priori assumptions are used when deducing these models (for instance, an assumption regarding hydrostatic equilibrium), or it is supposed that the observed intensities of the lines in the spot are caused to a great extent by diffused light from the photosphere. The opacity of these models is nearly the same as that of the photosphere.

Makita's model (1963) stands somewhat apart from these two types. It is a paper in which the assumption of a two-component structure of the sunspot umbra was first stated. The existence of two components is necessary to explain the radiation of both the lines Ti I and Fe I and the lines Fe II within the limits of a single model. For the radiation of Fe II lines physical conditions are necessary which would greatly differ from those generally excepted for the spot umbra and approaching those existing in the photosphere. Zwaan (1965) suggested that Fe II lines in the umbra are caused by diffused light from the photosphere, but the observed character of splitting of Fe II lines in the spot indicates that at least an essential part of the radiation in these lines goes directly from the spot (V. N. Obridko, 1968; E. I. Mogilevskij et al., 1967).

In spite of the fact that, in principle, it is possible to construct a homogeneous model allowing the Fe II radiation in the spot to be explained as being due to the radiation from deep layers (R. B. Teplickaja, 1967), a number of other experimental and theoretical facts show that some essential inhomogeneities must exist in the spot, and the hottest of them are close to those of the photosphere in their physical conditions. A number of investigators observed the existence of bright points within the spot umbra (R. Bray and R. Loughed, 1964; Beckers and Schröter, 1967) and in those cases when the intensity of these formations could be measured with confidence, it proved to be comparable with the photosphere brightness (Beckers and Schröter, 1967). Observations of the magnetic field in the spot show, as a rule, their great inhomogeneity (A. B. Severnyj 1965; B. A. Ioshpa and V. N. Obridko, 1965; I. A. Žulin et al., 1967; V. N. Obridko, 1968). After all, theoretical calculations of a possible structure for quasi-force-free field of the spot (F. A. Ermakov et al., 1967; E. I. Mogilevskij et al., 1967), as well as calculations of oscillatory convection in the spot (S. I. Syrovatskij and Yu. A. Žugžda, 1967), indicate the probable existence of fine-structure elements in the spot, which greatly differ in their physical properties.

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2. *Calculations of an inhomogeneous model of the spot-umbra*

It is of interest to consider the question of the possible changing of the sunspot umbra model if the inhomogeneities are taken into account. Let us suppose that the hot component is identical to patches of photosphere matter in the spot umbra and occupies less than 10% of the area. (M. Makita, 1963). The hot component is concentrated in the small elements comparable to (or smaller than) the resolution limit of modern observations. Apparently taking the inhomogeneities into account in opaque models will not greatly change these models. This would be due to the fact that these models differ little from the photosphere by their pressure, and 10% substitution of the umbra matter for the photosphere matter will not essentially affect the spot opacity (the temperature effect on the optical thickness is quite small). On the other hand, it is quite clear that the influence of inhomogeneities on rarefied models must be very great. This is associated with a non-linear dependence of the optical thickness τ on the gas pressure P_g. In particular, for example, at $\Theta = 1{\cdot}3$ and $P_g \sim 10^4$ (Allen, 1955)

$$d\tau = \varkappa \varrho \, dh \sim \varkappa \Theta P_g \, dh \sim \Theta P_g^{1\cdot6} .$$

Thus, small patches of thick photosphere matter should lead to a great increase of opacity.

As initial models for calculations, Michard (1953) and Fricke and Elsässer (1965) models of spots were taken with the corresponding models of the photosphere. The occupation of the hot component in the area was supposed to be equal to 0·10 or 0·05. The hot component was identified with the photosphere matter. The cold component temperature was chosen in such a way that in a continuous spectrum the total radiation from both components is in accordance with the observed values. (The Planck formula was taken for $\lambda = 5000$ Å). This led to some decrease of temperature (a very small one, as a rule) in the cold component as compared to the initial models. However, it was found that for a colder model by Fricke and Elsässer the occupation at 0·10 is hardly probable, because this would lead to unreal low values of temperature in the cold component. Therefore, when calculating the model on Fricke and Elsässer's data we used only one value, $\alpha = 0{\cdot}05$.

For the calculation of τ we supposed that the cold component pressure conforms with that computed in initial models. This was due to the fact that rarefied models were mainly deduced on the lines of neutral metals which are formed preferentially in the cold component. A part of radiation coming from the hot component can be estimated for various elements as follows. As soon as we have identified the hot component with photospheric patches, then we have for the part of the equivalent width of the line from the hot component $W_r^* = KW^*$

$$(2) \qquad W_r^* = KW^* = \frac{\alpha I^{\odot}}{I_c^*} = \alpha \frac{I_c^{\odot}}{I_c^*} W^{\odot} = \frac{\alpha}{b} \frac{I_c^{\odot}}{I_c^*} W^* .$$

Here b is the mean value of strengthening or weakening lines of the spot in the absence of magnetic intensification. We calculated the b-values in our previous work (V. N. Obridko, 1968). The values $K = \alpha/b \, . \, I_c^{\odot}/I_c^*$ are given in the Table I.

Table 1

Part of radiation in the lines of various elements originating in the hot component

	Michard		Fricke and Elsässer
	$\alpha = 0{\cdot}10$	$\alpha = 0{\cdot}05$	$\alpha = 0{\cdot}05$
Fe I	0·26	0·13	0·40
Ti I	0·12	0·06	0·20
Fe II	0·66	0·33	1·04
Ti II	0·44	0·22	0·71

In spite of the roughness of such an estimation one can see that the neutral element lines are mainly formed in the cold component.

In order to calculate the mean model it is supposed that the zero-points of the depths in hot and cold components are equal to each other. In Tables 2 and 3 the computed results are presented with the utilization of Michard's and Fricke-Elsässer's models respectively.*) Figures 1 and 2 show the dependence

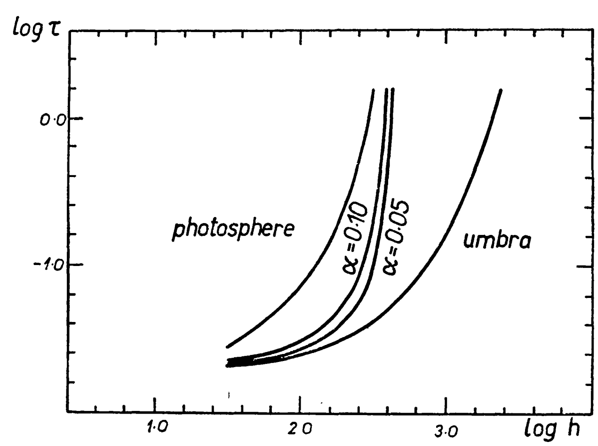

Fig. 1. The dependence of the optical depths in the spot umbra, in the photosphere and in the inhomogeneous model when Michard model was taken as initial one.

of the optical depths in the spot umbra, in the photosphere and in the inhomogeneous models for Michard and Fricke-Elsässer's initial models respectively.

Table 2

Model of the two-component umbra of the spot (initial model by Michard)

h km	$\log \bar{\tau}_0$	$\log \overline{P_g}$	Θ	hot $\log P_g$	hot Θ	cold $\log P_g$	cold Θ
$\alpha = 0{\cdot}10$							
0	−1·70	3·83	1·36	4·11	1·11	3·79	1·44
50	−1·63	3·89	1·35	4·28	1·08	3·81	1·43
100	−1·54	3·94	1·34	4·44	1·04	3·83	1·42
150	−1·42	4·05	1·32	4·59	0·99	3·85	1·39
200	−1·29	4·09	1·30	4·75	0·93	3·87	1·37
250	−1·12	4·16	1·26	4·88	0·86	3·89	1·32
300	−0·87	4·23	1·22	4·99	0·78	3·91	1·29
350	−0·41	4·31	1·12	5·10	0·64	3·93	1·19
400	−0·25	4·36	0·97	5·18	0·54	3·95	1·04
$\alpha = 0{\cdot}05$							
0	−1·70	3·81	1·36	4·11	1·11	3·79	1·39
50	−1·65	3·85	1·35	4·28	1·08	3·81	1·38
100	−1·57	3·89	1·34	4·44	1·04	3·83	1·37
150	−1·48	3·94	1·32	4·59	0·99	3·85	1·35
200	−1·39	3·99	1·30	4·75	0·93	3·87	1·33
250	−1·26	4·05	1·26	4·88	0·86	3·89	1·29
300	−1·06	4·10	1·22	4·99	0·78	3·91	1·25
350	−0·66	4·16	1·12	5·10	0·64	3·93	1·15
400	−0·04	4·22	0·97	5·18	0·54	3·95	1·00

Table 3

Model of the two-component spot-umbra (initial model by Fricke and Elsässer)

h km	$\log \bar{\tau}$	$\log P_g$	Θ	hot $\log P_g$	hot Θ	cold $\log P_g$	cold Θ
0·0	−2·00	3·84	1·57	4·29	1·075	3·80	1·82
50	−1·94	3·90	1·55	4·52	1.027	3·82	1·80
100	−1·85	3·96	1·54	4·74	0·977	3·83	1·79
150	−1·70	4·05	1·51	4·95	0·924	3·85	1·73
200	−1·48	4·12	1·47	5·11	0·848	3·86	1·67
250	−1·18	4·20	1·43	5·23	0·726	3·88	1·63
300	−0·82	4·24	1·37	5·30	0·598	3·90	1·53
350	−0·45	4·29	1·31	(5·37)	(0·525)	3·91	1·51
400	−0·05	4·33	1·21	(5·42)	(0·475)	3·93	1·37
450	−0·36	4·38	1·05	(5·48)	(0·435)	3·94	1·15

*) Table 3 shows in brackets the extrapolated values taking into account the Böhm-Vitense hydrogen convective zone (1954).

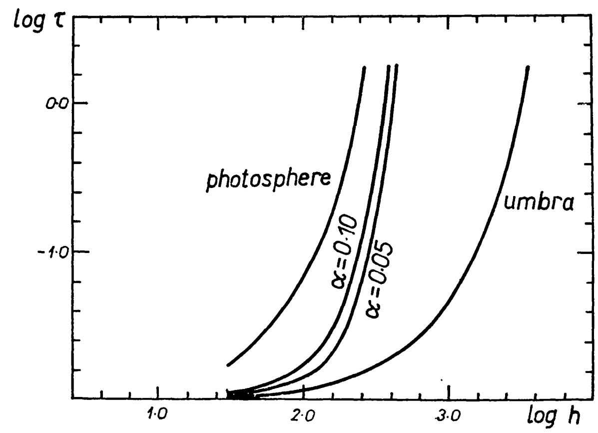

Fig. 2. The dependence of the optical depths in the spot umbra, in the photosphere and in the inhomogeneous model when Fricke and Elsässer model was taken as initial one.

3. *Discussion of the results*

1) One can see from Figs. 1 and 2 that if we take into account the photosphere patches, then some very considerable changes in the spot-umbra model will be caused. If previously the optical depth $\tau^* \sim 1$ was reached at $h \sim 2000-3000$ km, then now the spot opacity is comparable with the photosphere opacity and $\tau^* \sim 1$ corresponds to the depths h at 400 km only. It should be noted that the results depend little on the accepted value of the occupation α. Just 2−5% of the patches of photospheric matter is enough to obtain the great spot-umbra opacity.

2) A comparison of the results shown in Tables 2 and 3 with original models by Michard and Fricke-Elsässer show little difference between the mean model and initial models. Mean models were constructed in such a way that the Θ values coincided with the initial ones, the mean values P_g in the upper layers practically coincided while in the lowest layers they differed not more than twice (log P_g is equal to 4·22, and 3·96 respectively). Thus, the mean model within observational errors does not differ from rarefied models, though the difference in optical depths is very great. Thus, taking into account the inhomogeneities, one can, in principle, build, *a rarefied on average but opaque model,* thus removing existing objections against rarefied models.

3) Strong deviations from hydrostatic equilibrium are a matter of some difficulty in rarefied models. Thus, for Fricke and Elsässer's original model the effective acceleration of gravity will be

$$g_{\text{eff}} = \frac{\mathrm{d} \log P_g}{\mathrm{d}h} \frac{RT}{\mu\, 0{\cdot}43} \approx 0{\cdot}1\, g_\odot\,. \tag{3}$$

This difficulty may be decreased by taking homogeneities into account. In our model $g_{\text{eff}} \approx 0{\cdot}4\, g_{\odot}$. This is due to the fact that with the decrease of characteristic scale of the depths the value $\mathrm{d} \log P_g/\mathrm{d}h$ sharply increases, some 4–5 times, while the rest of the values remains constant.

4. *Some remarks*

When calculating models some assumptions were made, both evident and non-evident. In our conclusion we shall pay attention to them once more.

1) It was supposed that the hot component is fully identical to the photosphere patches. Apparently, this assumption is not quite correct. The results of magnetic field measurements in Fe II-lines show that there is a rather considerable field in the hot component (V. N. Obridko, 1968). Apparently, the identification of P_g distribution with geometrical depth in the cold component and analogous distribution in initial rarefied models is not quite correct either. Indeed, the geometrical dependence in each component, i.e. the transfer from $P_g(\tau)$ to $\tau(h)$ in each component, must be performed by taking the existence of inhomogeneities into account. Moreover when calculating we did not take into account small differences between Θ in the cold component and in the initial models.

2) When calculating τ and P_g, in order to simplify computation, the zero-points of depths in both components were considered to be identical and equal to the depth zero-point of the photosphere. Apparently, this assumption is far from reality and indeed the spot should be some sort of depression in the photosphere.

3) When computing the continuous radiation from the spot we supposed the following formula to be valid

$$\bar{B}^* = \alpha B^{\odot} + (1 - \alpha) B^c , \tag{4}$$

where B – Planck function for corresponding temperature, $B^{\odot}$ – hot component radiation, B^c – cold component radiation.

When computing the optical depth an analogous formula was used:

$$\bar{\tau}^* = \alpha \tau^{\odot} + (1 - \alpha) \tau^c , \tag{5}$$

where $\tau^{\odot}$-optical depth in the hot component, τ^c-optical depth in the cold component. Indeed, neither of these formulae is quite exact and the light propagation in the medium consisting of translucent hot and cold elements is a difficult problem of the three-dimensional theory of transfer. In such a medium it is necessary to take into account one more effect – light diffusion on large-scale atmospheric inhomogeneities. This effect must cause the redistribution of the brightness among the components in the spot and an additional decrease of the spot-umbra opacity. However, a strict calculation of all the above effects is very difficult.

Thus, the model obtained in our work is only a preliminary one. It illustrates only the *possibilities* appearing when inhomogeneities are taken into account. Nevertheless, even this preliminary model is free of many defects inherent to homogeneous models.

One may hope that even this preliminary model will nevertheless be nearer to reality than homogeneous models.

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