

Geomagnetic Field Perturbations Resulted from Tsunami Wave Impact on the Ionosphere

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Abstract—The generation mechanism of the geomagnetic field perturbations associated with tsunami wave propagation in ocean is examined. The geomagnetic perturbations are produced by electric currents generated in both the seawater and conductive layers of the ionosphere. The electric current in conductive seawater is caused by the seawater motion due to tsunami wave propagation whereas the current in the ionospheric plasma is generated by acoustic gravity wave (AGW) incident on the ionosphere from the atmosphere. The AGW is originated from vertical displacements of seawater surface due to the tsunami wave propagation. Although the ionospheric plasma conductivity is much lower than the seawater conductivity, the electric current in the ionosphere can be greater than that in the seawater due to an exponential increase of amplitude of the upward-propagating AGW. Our calculations are indicative of the possibility of space monitoring of tsunami wave based on onboard measurements of the geomagnetic field perturbations.

1. INTRODUCTION

A possibility of the tsunami wave detection by using the monitoring of the ionospheric perturbations was first discussed by Peltier and Hines [1]. The excitation of the ionospheric disturbances was assumed to be due to tsunami-induced acoustic gravity wave (AGW) propagating upward in the atmosphere from the ocean surface up to the ionospheric altitudes. The technique of the ionosphere probing and network of GPS receivers have been used by Artru et al. [2] to detect the ionospheric perturbations caused by AGW propagation. The peculiarities of internal gravity waves (IGW) produced by tsunami waves in the neutral atmosphere and interaction of the IGW with the ionospheric plasma have been studied by Occhipinti et al. [3]. A net of GPS receivers arranged at Hawaii islands has been used to detect the changes in total electron content (TEC) possibly associated with earthquakes occurrence [4]. The TEC variations in the ionosphere were observed after earthquakes at the Kuril Islands on 2006 and Samoa Island on 2009 as well as at Chili on 2010. This effect is presumed to be related to the displacement of seawater surface caused by tsunami wave propagation. The ionospheric perturbations have been observed during Tohoku-Oki earthquake and tsunami of March 11, 2011 [5]. The TEC variations in the ionosphere were recorded by using a net of GPS receivers in Japan. These ionospheric perturbations were found to propagate from the earthquake epicenter at the velocities of 3.4, 1.0 and 0.2–0.3 km/s. The ionospheric perturbations possibly related to tsunami wave have been observed near Tohoku coast over the Pacific Ocean [6]. The data analysis of the ionospheric TEC perturbations has indicated that the typical wavelength/vertical scale of the tsunami-induced AGW is about 50 km while the perturbation region covers the altitude interval of the order of 200 km.

The tsunami-driven motion of the conductive seawater immersed in the geomagnetic field gives rise to the generation of electric current in the seawater, which in turn results in the geomagnetic

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perturbations (e.g., see [7]). The electromagnetic effect produced by tsunami can be detected on the seacoast and ocean floor. Toh et al. [8] have reported evidence on the geomagnetic perturbations associated with two tsunami waves originated from two earthquakes which occurred at the Kuril Islands on November 15, 2006 and on January 13, 2007, respectively. The measurements at a seafloor geomagnetic observatory have shown that the horizontal component of the geomagnetic perturbations had opposite signs during growth and decay of wave. Manoj et al. [9] have reported the ground-based observations of the geomagnetic perturbations of 1 nT amplitude caused by tsunami wave after the strong Chilean earthquake on February 27, 2010. Wang and Liu [10] have studied the geomagnetic field variations induced by tsunami waves in open ocean. The theoretical analysis shows that the tsunami wave of 1 m height causes the magnetic perturbation of the order of 10 nT on the sea surface. The amplitude of the perturbation depends on both the wave parameters and orientation of the Earth's magnetic field.

The tsunami-produced geomagnetic perturbations are caused by the electric currents generated in both the seawater and the ionosphere. As we have noted above, the tsunami-driven motion of the conductive seawater gives rise to the electric current in the seawater whereas the current and geomagnetic perturbations in the ionospheric plasma are generated by the AGW propagating from the atmosphere into the ionosphere. The AGW in the atmosphere originates from the vertical displacements of seawater surface due to tsunami wave propagation. Then AGW propagates upward and penetrates into the ionosphere thereby exciting the ionospheric plasma motion. Sorokin and Fedorovich [11] have shown that both the ionospheric plasma motion and the changes in plasma ionization in the area of ionospheric current flow results in the generation of the geomagnetic field perturbations.

The shape of tsunami wave is determined by the time dependence of sea floor displacement due to subsea earthquake. In this study we examine the geomagnetic perturbations generated by tsunami waves, which in turn result from different type of the earthquakes. An idealized model of the medium with vertical geomagnetic field is constructed in order to avoid complicated expressions and to estimate the contribution to the total geomagnetic perturbation from the currents flowing in seawater and in the ionosphere. Our main concern is to demonstrate that, first, the tsunami wave propagation away from epicenter is accompanied by the generation of electric currents both in the sea medium and in the ionosphere; second, the ionospheric current gives rise to the magnetic field comparable to the field originated from the current in the sea medium. At such distance the tsunami wave can be approximated as a locally one-dimensional wave in linear approximation while all the physical variables can be treated as stationary fields propagating at the tsunami speed. For example, such approach has been used by Wei et al. [12]. The model makes it possible to analyze the dependence of the geomagnetic perturbations on medium parameters and to estimate the effectiveness of different mechanisms for the generation of these perturbations.

2. ACOUSTIC GRAVITY WAVE CAUSED BY VERTICAL DISPLACEMENT OF SEAWATER SURFACE

Tsunami waves are long surface gravity waves propagating in the sea medium without dispersion. The velocity of gravity waves is $a = (gh)^{1/2}$, where h denotes sea depth and g is free fall acceleration. A Cartesian system is adopted with z -axis directed vertically upward. The origin of the coordinate system is situated on the sea surface. The direction of tsunami wave propagation is positive parallel to horizontal x -axis. The tsunami wave is assumed to be due to the seabed displacement caused by subsea earthquake. The earthquakes can be split into two types depending on whether they have residual displacement of the seabed or not. The first type is referred to as the class of “piston” earthquakes while the other type is referred to as “membrane” earthquakes [13, 14]. The main property of the “piston” seismic source is the fast vertical displacement of sea floor followed by the generation of a surface single-polarity gravity wave and by the formation of residual sea floor displacement. The source of “membrane” type does not cause the residual displacement; that is, at the beginning of the seismic shock the seabed ascends and then it descends to its original position. Such a source generates a surface bidirectional gravity wave. In the long waves approximation the tsunami wave propagates without dispersion. Thus for the seismic source of the first type we choose the following spatiotemporal dependence of the sea surface vertical

displacement due to tsunami wave:

$$\xi_p(x, t) = \frac{\xi_{0p}}{\cosh^2 \chi(x - at)},$$

whereas for the second type of sources the vertical displacement is assumed to be

$$\xi_m(x, t) = \xi_{0m} \left[\frac{1}{\cosh^2 \chi(x - at - x_m)} - \frac{1}{\cosh^2 \chi(x - at + x_m)} \right],$$

where ξ_{0p} , ξ_{0m} , χ , and x_m are given constants. The vertical displacement of the sea surface as a function of horizontal coordinate is shown in Figure 1 for both types of tsunami sources.

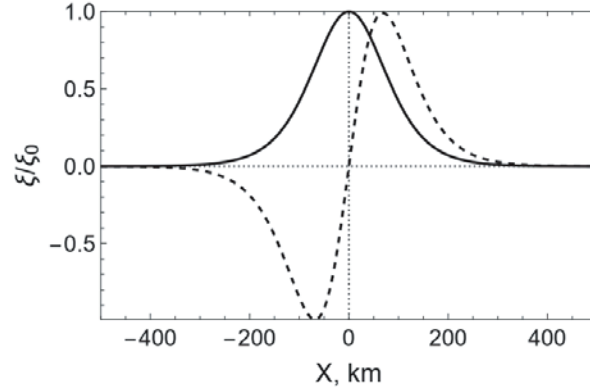


Figure 1. Vertical displacement of seawater surface caused by “piston” (solid line) and “membrane” (dashed line) seismic underwater sources.

Applying Fourier transform to the above equations we get:

$$\begin{aligned} \xi_{p,m}(k, \omega) &= 2\pi\xi_{0p,m}G_{p,m}(k)\delta(\omega - ka) \\ G_p(k) &= \frac{\pi k}{\chi^2 \sinh(\pi k/2\chi)}; \quad G_m(k) = -2i \sin(kx_m) G_p(k) \end{aligned} \quad (1)$$

where ω is the wave frequency, k the wave number, $\delta(x)$ denotes Dirac delta function, and $\xi_{0p,m}$ the amplitude of the sea surface vertical displacement. Here the inferior indices p and m correspond to the waves originated from the sources of “piston” and “membrane” types.

At first, let us study the main properties of the atmospheric AGW generated by the sea surface displacement due to tsunami wave propagation. Consider a model of the isothermal strato-heterogeneous atmosphere in the absence of wind. In linear approximation, the equation describing a distribution of gas velocity $\mathbf{v}(\mathbf{r}, t)$ in the AGW can be written as [15, 16]:

$$\frac{\partial^2 \mathbf{v}}{\partial t^2} = \nabla (c^2 \nabla \cdot \mathbf{v} + \mathbf{g} \cdot \mathbf{v}) + (\gamma - 1) \mathbf{g} (\nabla \cdot \mathbf{v}), \quad (2)$$

where $\gamma = 1.4$ is the adiabatic exponent; $c = (\gamma p_0 / \rho_0)^{1/2} = \text{const}$ is the sound velocity in the atmosphere; p_0 and ρ_0 are the undisturbed pressure and density of the atmospheric gas; $\mathbf{g} = -g\mathbf{e}_z$ where \mathbf{e}_z is the upward-directed unit vector. If all the functions is assumed to vary with coordinate x and time as $\exp(ikx - i\omega t)$, then Equation (2) reduces to the following system for vertical v_z and horizontal v_x components of the gas velocity:

$$\frac{d^2 v_z}{dz^2} - \frac{1}{H} \frac{dv_z}{dz} + \left[\frac{\omega^2}{c^2} + k^2 \left(\frac{\omega_g^2}{\omega^2} - 1 \right) \right] v_z = 0; \quad v_x = \frac{ik}{k^2 - \omega^2/c^2} \left(\frac{dv_z}{dz} - \frac{v_z}{\gamma H} \right), \quad (3)$$

where the constant $H = c^2/\gamma g$ stands for the height of homogeneous atmosphere, and $\omega_g = g(\gamma - 1)^{1/2}/c$ is the Brunt-Väisälä frequency. Equation (3) should be supplemented by the proper boundary

conditions. First, we suppose that the vertical component of the AGW-induced air velocity is a given function at the sea surface level $z = 0$. The other requirement on the gas velocity is that the vertical component of energy flux density averaged on x and t has to be positive in the infinity. The implication here is that on the average, the wave energy has to be transferred upward from the sea surface into the atmosphere. The solution of Equation (3) under the above conditions is found in Appendix A. The components of the AGW-induced air velocity are given by

$$\begin{aligned} v_z(k, z, \omega) &= v_z(k, 0, \omega) \exp\left(iKz + \frac{z}{2H}\right); \\ v_x(k, z, \omega) &= -iv_z(k, 0, \omega) \frac{k(\Gamma - iK)}{k^2 - \omega^2/c^2} \exp\left(iKz + \frac{z}{2H}\right); \\ \Gamma &= \frac{2 - \gamma}{2\gamma H}; \quad q = \left[\frac{\omega^2}{c^2} + k^2 \left(\frac{\omega_g^2}{\omega^2} - 1 \right) - \frac{1}{4H^2} \right]^{1/2}; \\ K &= -\text{sgn}(\omega) q(k, \omega); \end{aligned} \quad (4)$$

where $v_z(k, 0, \omega)$ is the vertical component of the air velocity at sea level. The latter value must coincide with the vertical component $V_z(k, 0, \omega)$ of the fluid velocity generated by the tsunami wave at the sea surface; that is:

$$v_z(k, 0, \omega) = V_z(k, 0, \omega). \quad (5)$$

Now we express the fluid velocity components $V_x(k, 0, \omega)$ and $V_z(k, 0, \omega)$ through the tsunami-induced vertical displacement $\xi_{p,m}(k, \omega)$ of the seawater surface. Taking into account that $V_{zp,m}(x, 0, t) = \partial \xi_{p,m}(x, t) / \partial t$, we obtain $V_{zp,m}(k, 0, \omega) = -i\omega \xi_{p,m}(k, \omega)$. The horizontal component of the sea medium velocity can be derived under the requirement that the fluid is incompressible; that is, $\nabla \cdot \mathbf{V} = 0$. The latter equation reduces to $ikV_x(k, z, \omega) = -dV_z(k, z, \omega)/dz$. Theoretical analysis of the tsunami wave propagation made by Pelinovsky [17] has shown that in the shallow water approximation, $kh \ll 1$, and under requirement of small displacement of the sea surface, $\xi/h \ll 1$, the horizontal component of the fluid velocity is independent of depth. Hence, the vertical component of the fluid velocity is linear with the depth $V_z(k, z, \omega) = V_z(k, 0, \omega)(1 + z/h)$. As a result, we obtain the sought relation between the fluid velocity components at $z = 0$ and the sea surface displacement in the tsunami wave:

$$V_{xp,m}(k, 0, \omega) = \omega \xi_{p,m}(k, \omega) / kh; \quad V_{zp,m}(k, 0, \omega) = -i\omega \xi_{p,m}(k, \omega). \quad (6)$$

Combining Equations (4), (5), and (6), we can find the gas velocity distribution in the ionosphere resulted from the AGW propagation. In particular, the horizontal component of the gas velocity at the altitude $z = z_1$ is given by

$$v_{xp,m}(k, z_1, \omega) = -\xi_{p,m}(k, \omega) \frac{\omega k(\Gamma - iK)}{k^2 - \omega^2/c^2} \exp\left(\frac{z_1}{2H} + iKz_1\right). \quad (7)$$

Substituting Equation (1) for $\xi_{p,m}(k, \omega)$ into Equations (4)–(6) and then applying inverse Fourier transform one can derive a spatiotemporal distribution of the gas velocity in the tsunami-induced AGW. Taking into account the sign of K , integration over ω and k gives

$$\begin{aligned} v_{x,z}^{(p,m)}(x, z, t) &= \frac{1}{\pi} \text{Re} \int_0^\infty \bar{v}_{x,z}^{(p,m)}(k, z) \exp[ik(x - at)] dk \\ \bar{v}_x^{(p,m)}(k, z) &= -a \xi_{0p,m} G_{p,m}(k) \frac{\Gamma + iq}{1 - h/(\gamma H)} \exp\left(\frac{z}{2H} - iqz\right); \\ \bar{v}_z^{(p,m)}(k, z) &= -ia \xi_{0p,m} k G_{p,m}(k) \exp\left(\frac{z}{2H} - iqz\right) \end{aligned} \quad (8)$$

As before, the indices p and m correspond to the ‘‘piston’’ and ‘‘membrane’’ seismic sources.

In what follows Equations (1) and (8) are used as the base for study of AGW propagation in the atmosphere. The numerical calculations of the functions $W_{p,m}(x, z) = v_z^{(p,m)}(x, z) \exp(-z/2H)$ taken in vertical plane (x, z) at the time $t = x/a$ are shown in Figure 2. These functions describe the distribution

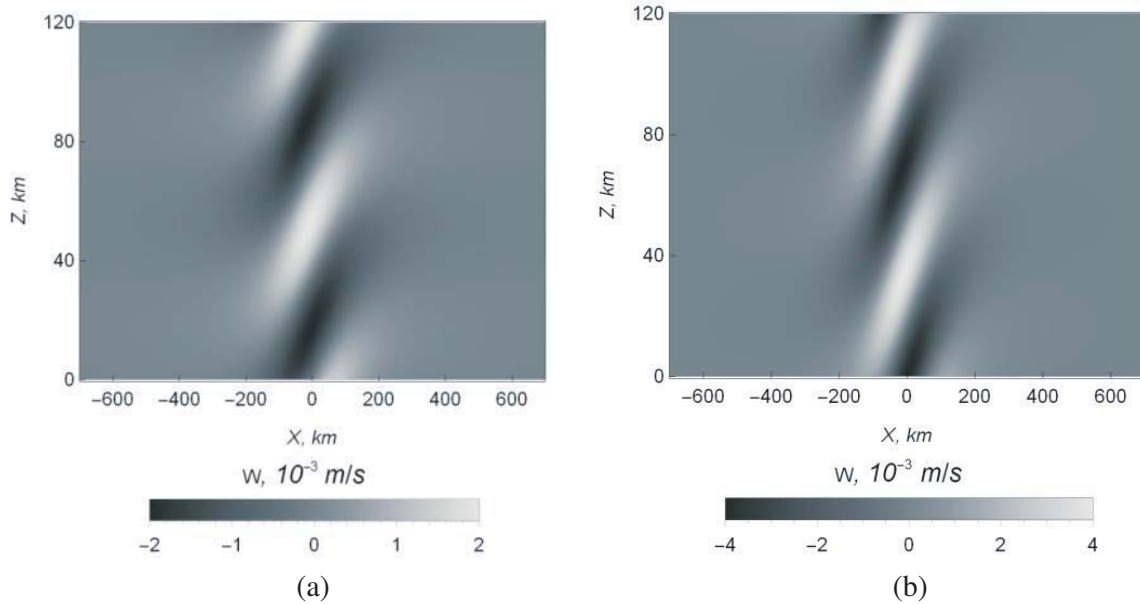


Figure 2. Numerical modelling of spatial distributions of functions (a) $W_p(x, z)$ and (b) $W_m(x, z)$ in vertical plane in the atmosphere. The numerical values $x_0 = 100$ km and $h = 3$ km are used.

of the vertical gas velocity without considering the exponential factor $\exp(-z/2H)$. It obvious from Figure 2 that AGW propagates upward in the atmosphere over the area of tsunami wave propagation. The amplitude of the vertical velocity can reach a value about 10 m/s.

3. EQUATIONS OF ELECTROMAGNETIC FIELD

In this section, we study the geomagnetic perturbations generated by tsunami wave. A model of the tsunami-driven electric currents is sketched in Figure 3. The electric currents generated in the sea medium and the ionosphere are shown with lines 6 and 9 on the left side of this figure. The ionospheric current 9 are closed through the field-aligned current 8 and then through the current in conjugate ionosphere shown with line 9 on the right side of Figure 3. In this model the undisturbed

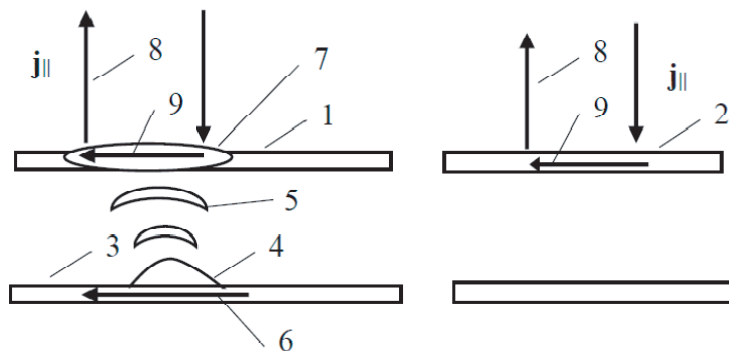


Figure 3. A schematic plot of the generation of current system. (1) A conducting layer in lower ionosphere; (2) a conducting layer in conjugate ionosphere; (3) sea medium; (4) tsunami wave; (5) acoustic gravity wave; (6) electric current in the sea medium; (7) area of perturbation in the ionosphere; (8) field-aligned current in the upper ionosphere and magnetosphere; (9) electric current in the ionosphere and conjugate ionosphere.

geomagnetic field \mathbf{B}_0 is pointed upward. The sea medium with constant conductivity σ_w occupies the layer $-h < z < 0$. Let $\hat{\sigma}$ be the conductivity tensor of the thin conductive gyrotropic layer of the ionosphere. Maximum of this ionospheric layer is located at the altitude $z = z_1$. The magnetosphere is situated above the ionospheric layer. In the magnetosphere the conductivity component perpendicular to the geomagnetic field lines is assumed to be zero. The nonconducting atmosphere is sandwiched between the sea medium and the ionosphere. The solid rock occupies half space $z < -h$ below the sea floor. The rock conductivity is presumed to be negligible compared with the seawater conductivity.

The motion of conductive medium in the geomagnetic field results in the generation of electric current and geomagnetic perturbation $\mathbf{b} = \mathbf{B} - \mathbf{B}_0$. In the frequency range of interest the displacement current can be disregarded since its value is negligible compared with the conduction current. Thus, Maxwell's equations for the ionosphere and seawater read

$$\nabla \times \mathbf{b} = \mu_0 \mathbf{j}; \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{b}}{\partial t}; \quad \nabla \cdot \mathbf{b} = 0, \quad (9)$$

where \mathbf{E} is the electric field, and μ_0 is the magnetic permeability of free space/magnetic constant. The conduction current density \mathbf{j} can be found from Ohm's law for moving conductive media. Taking into account that the magnetic perturbation is small; that is, $b \ll B_0$, then Ohm's law for the seawater reduces to

$$\mathbf{j} = \sigma_w (\mathbf{E} + \mathbf{V} \times \mathbf{B}_0), \quad (10)$$

where $\mathbf{V}(\mathbf{r}, t)$ is the vector field of mass velocities generated by tsunami wave in the seawater. We shall consider the case of tsunami wave propagating in (x, z) plane whence it follows that $\partial/\partial y = 0$. Rearranging Maxwell's equations (9) and Ohm's law (10), we obtain the following equation for the electric field component E_y in the seawater:

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} - \mu_0 \sigma_w \frac{\partial E_y}{\partial t} = -\mu_0 \sigma_w B_0 \frac{\partial V_x}{\partial t} \quad (11)$$

The depth of skin layer in the seawater can be estimated as $\delta_s = (\tau/2\pi\mu_0\sigma_w)^{1/2}$ where τ is the typical period of tsunami wave. Substituting $\tau \approx 10$ min and the seawater conductivity $\sigma_w = 3$ S/m into the above relationship we get the value $\delta_s \approx 10$ km, which is greater than the sea depth. Consequently, the electric field in the seawater weakly depends on the depth z . Furthermore, the horizontal component of the fluid velocity in tsunami wave is practically constant in depth z . In this notation, in order to derive the boundary condition for the component E_y that relates the horizontal electric field just below and above thin conductive layer of the sea medium, one can formally integrate Equation (11) over the coordinate z from $-h$ to zero under requirements that $h \rightarrow 0$ and $\sigma_w h = \text{const}$. As a result we obtain:

$$\left\{ \frac{\partial E_y}{\partial z} \right\}_{z=0} - \mu_0 \Sigma_w \frac{\partial E_y}{\partial t} = -\mu_0 \Sigma_w B_0 \frac{\partial}{\partial t} V_x(x, 0, t); \quad \{E_y\}_{z=0} = 0, \quad (12)$$

where $\Sigma_w = \sigma_w h$ is the depth integrated conductivity of the seawater, and the braces $\{\dots\}$ denotes the function discontinuity/jump; that is, difference between values of the function taken on the upper and lower surfaces of the conductive layer.

Consider now the ionospheric perturbations caused by the upward propagating AGW, which in turn results from the tsunami-driven motion of the sea surface. The Ohm's law in the ionosphere is given by

$$\mathbf{j} = \hat{\sigma} (\mathbf{E} + \mathbf{v} \times \mathbf{B}_0); \quad \hat{\sigma} = \begin{pmatrix} \sigma_P & \sigma_H & 0 \\ -\sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_{\parallel} \end{pmatrix}, \quad (13)$$

where σ_{\parallel} is the field-aligned plasma conductivity; σ_P and σ_H are the Pedersen and Hall conductivities, respectively; $\mathbf{v}(\mathbf{r}, t)$ is the gas velocity in the AGW propagating through the ionosphere. At the ionospheric altitudes the field-aligned conductivity is much greater than σ_P and σ_H . Assuming that $\sigma_{\parallel} \rightarrow \infty$, the parallel electric field thus becomes infinitesimal, i.e., $E_z = 0$. Substituting Ohm's law in Eq. (13) into Maxwell equations (9), we come to an equation for the horizontal electric field in the ionosphere:

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} - \mu_0 \left(-\sigma_H \frac{\partial E_x}{\partial t} + \sigma_P \frac{\partial E_y}{\partial t} \right) = -\mu_0 \sigma_P B_0 \frac{\partial v_x}{\partial t} \quad (14)$$

The Pedersen and Hall conductivities are assumed to be not zeroth in the layer of ~ 30 km thickness around the altitude ~ 120 km where they reach a peak value. The skin depth for this ionospheric layer can be estimated as $\delta_s \approx (\tau/2\pi\mu_0\sigma_{P,H})^{1/2}$. Substituting typical values $\sigma_{P,H} \approx 3 \times 10^{-4}$ S/m for the daytime ionospheric conductivities into the above relationship, we obtain $\delta_s \approx 10^3$ km, which is much greater than the thickness of the ionospheric conductive layer. This means that the horizontal component of the electric field in this layer practically does not change in vertical direction. It makes possible to replace Equation (14) with the boundary conditions for the transition through the thin conductive ionospheric layer. To do this one should formally replace the altitude dependence of the ionospheric conductivities by delta function; that is $\sigma_{P,H} = \Sigma_{P,H}\delta(z - z_1)$ where the point $z = z_1$ corresponds to the point of the conductivities maximum. Integrating Equation (14) over z and using these formulae for the ionospheric conductivities, we obtain the boundary condition for the ionosphere in the thin-layer approximation

$$\left\{ \frac{\partial E_y}{\partial z} \right\}_{z=z_1} - \mu_0 \frac{\partial}{\partial t} (-\Sigma_H E_x + \Sigma_P E_y) = -\mu_0 B_0 \Sigma_P \frac{\partial}{\partial t} v_x(x, z_1, t); \quad \{E_y\}_{z=z_1} = 0. \quad (15)$$

Here the use is made of the height-integrated Pedersen and Hall conductivities:

$$\Sigma_{P,H} = \int \sigma_{P,H}(z) dz.$$

Now consider the continuity equation $\nabla \cdot \mathbf{j} = 0$ for electric current density in the ionosphere. Integrating this equation with respect to z across the conductive layer of the ionosphere yields

$$\frac{\partial}{\partial x} [\Sigma_P E_x + \Sigma_H E_y - \Sigma_H B_0 v_x(x, z_1, t)] = -[j_z(z_1 + 0) - j_z(z_1 - 0)] = -j_{\parallel}, \quad (16)$$

where $j_{\parallel} = j_z(z_1 + 0)$ is the field-aligned current flowing from the ionosphere into the magnetosphere. Here we take into account that the current $j_z(z_1 - 0)$ at lower boundary of the ionosphere is equal to zero. Owing to the high value of the longitudinal conductivity σ_{\parallel} , the field-aligned current in the magnetosphere has to be closed to the conductive layer of the conjugate ionosphere as shown in Figure 3. In quasistatic approximation the electric field in the magnetosphere can be found from the second equation of the equation set (9). Applying curl operator to this equation, we obtain $\nabla \times (\nabla \times \mathbf{E}) = 0$. The components of this vector equation can be written as

$$\frac{\partial^2 E_x}{\partial z^2} = 0; \quad \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} = 0$$

It follows from these equations that the component E_x does not change when transferring along the geomagnetic field lines into the conjugate ionosphere whereas the component E_y decreases with distance from the ionosphere with the typical damping scale of the order of tsunami wave size; that is, of the order of hundreds kilometers. This means that the component E_y vanishes before it reaches the conjugate ionosphere. Taking into account that the field aligned current is transferred into the conjugate ionosphere without changes, Equation (16) for this region is reduced to

$$\frac{\partial}{\partial x} \Sigma_P E_x = j_{\parallel}. \quad (17)$$

Adding and subtracting Equations (16) and (17), we obtain

$$E_x = -\frac{\Sigma_H}{2\Sigma_P} [E_y - v_x(x, z_1, t) B_0]; \quad j_{\parallel} = -\frac{\Sigma_H}{2} \frac{\partial}{\partial x} [E_y - v_x(x, z_1, t) B_0] \quad (18)$$

In order to rearrange boundary conditions (12) and (15) for the electric field components in the sea medium and the ionosphere, we substitute Equation (18) for E_x into Equation (15). Taking into account that all perturbed quantities vary as $\exp(ikx - i\omega t)$ and rearranging we come to the following boundary conditions for the component E_y

$$\begin{aligned} \left\{ \frac{dE_y}{dz} \right\}_{z=z_1} + i\omega\mu_0\Sigma_c E_y(z_1) &= i\omega\mu_0\Sigma_c B_0 v_x(k, z_1, \omega); \quad \{E_y\}_{z=z_1} = 0 \\ \left\{ \frac{dE_y}{dz} \right\}_{z=0} + i\omega\mu_0\Sigma_w E_y(0) &= i\omega\mu_0\Sigma_w B_0 V_x(k, 0, \omega); \quad \{E_y\}_{z=0} = 0 \end{aligned} \quad (19)$$

$$\Sigma_c = \Sigma_P + \Sigma_H^2/2\Sigma_P$$

The electric field in the atmosphere and seabed rocks is determined by Equation (9) in which $\mathbf{j} = 0$. The electric field in the upper ionosphere and magnetosphere is determined by Equations (9) and (13) in which $\mathbf{v} = 0$ and $\sigma_P = \sigma_H = 0$. Equation describing the component E_y in all the media is given by

$$\frac{d^2 E_y}{dz^2} - k^2 E_y = 0.$$

If the solution of this equation under boundary requirement (19) is known, then other components of the geomagnetic perturbation can be found from the second equation of the equation set (9):

$$b_x = \frac{i}{\omega} \frac{dE_y}{dz}; \quad b_z = \frac{k}{\omega} E_y.$$

As a result, we obtain the geomagnetic field perturbation in the atmospheric layer $0 < z < z_1$

$$\begin{aligned} b_x(k, \omega, z) &= B_0 [v_x(k, \omega, z_1) F_x^i(k, \omega, z) - V_x(k, \omega) F_x^w(k, \omega, z)] \\ b_z(k, \omega, z) &= -iB_0 [v_x(k, \omega, z_1) F_z^i(k, \omega, z) + V_x(k, \omega) F_z^w(k, \omega, z)] \end{aligned} \quad (20)$$

where $v_x(k, \omega, z_1)$ is the horizontal component of the gas velocity in the AGW at the altitude of the ionosphere, and $V_x(k, \omega)$ is the horizontal component the seawater velocity in the tsunami wave. Here we make use of the following abbreviations:

$$\begin{aligned} F_{x,z}^i(k, \omega, z) &= \frac{u_w e^{kz} \mp i(\omega/k) e^{-kz}}{u_i u_w e^{kz_1} + (\omega/k)^2 e^{-kz_1}}; \quad F_{x,z}^w(k, \omega, z) = \frac{u_i e^{k(z_1-z)} \mp i(\omega/k) e^{-k(z_1-z)}}{u_i u_w e^{kz_1} + (\omega/k)^2 e^{-kz_1}}, \\ u_i &= c_i - i\omega/k; \quad u_w = c_w - i\omega/k; \quad c_i = 2/\mu_0 \Sigma_c; \quad c_w = 2/\mu_0 \Sigma_w \end{aligned} \quad (21)$$

where the signs $(-)$ and $(+)$ correspond to x and z components, respectively.

Setting $\Sigma_c = 0$ and $c_i \rightarrow \infty$ we can examine the case where the influence of the ionosphere is insignificant. Substituting these values into Equations (20) and (21), we obtain the magnetic field perturbation generated by the tsunami wave for this extreme case:

$$b_x(k, \omega, z) = -B_0 \frac{k \exp(-kz)}{kc_w - i\omega} V_x(k, \omega).$$

This equation coincides with the formula derived by Wang and Liu [10] for the model in which the geomagnetic perturbation originates from the tsunami driven motion of the seawater.

4. PERTURBATION OF GEOMAGNETIC FIELD IN THE ATMOSPHERE

In order to study the geomagnetic field in the atmosphere we first substitute Equations (1), (6), and (7) for $\xi_{p,m}$, $V_{xp,m}$, $V_{zp,m}$ and $v_{xp,m}$ into Equation (20). Then applying the inverse Fourier transform, taking into account the sign of K and performing integration over ω and k , we obtain the spatiotemporal distribution of the geomagnetic field perturbation in the atmosphere:

$$\begin{aligned} b_{x,z}^{(p,m)}(x, z, t) &= \frac{1}{\pi} \text{Re} \int_0^\infty \bar{b}_{x,z}^{(p,m)}(k, z) \exp[ik(x - at)] dk \\ \bar{b}_{x,z}^{(p,m)}(k, z) &= -B_0 \frac{\xi_{p,m}}{h} a G_{p,m}(k) \left[\frac{h(\Gamma + iq)}{1 - h/\gamma H} \exp\left(\frac{z_1}{2H} - iqz_1\right) F_x^i(k, ka, z) + F_x^w(k, ka, z) \right] \\ \bar{b}_{x,z}^{(p,m)}(k, z) &= -iB_0 \frac{\xi_{p,m}}{h} a G_{p,m}(k) \left[-\frac{h(\Gamma + iq)}{1 - h/\gamma H} \exp\left(\frac{z_1}{2H} - iqz_1\right) F_z^i(k, ka, z) + F_z^w(k, ka, z) \right] \end{aligned} \quad (22)$$

It should be noted that the inequality $1 - h/\gamma H > 0$ takes place in both the atmosphere and sea medium. The implication here is that the velocity of tsunami wave is smaller than the sound speed in the atmosphere.

The height-integrated ionospheric conductivities can vary within broad range $\Sigma_{P,H} \sim 0.1 \div \geq 10$ S (e.g., Kelley [18]). In order to estimate the parameters appearing in Equation (21) we use the numerical values of $\Sigma_c = 10$ S which is typical for weakly disturbed daytime ionosphere. The seawater conductivity

changes from 1 to 5 S/m depending on water area, salinity, water temperature and etc. Following Wang and Liu [10] we choose the value $\sigma_w = 4$ S/m. Substituting the above values as well as the following numerical values $h = 10^3$ m and $\Sigma_w = 4 \times 10^3$ S into Equation (21) to yield $c_i \approx 2 \times 10^5$ m/s, $a \approx 10^2$ m/s and $c_w \approx 4 \times 10^2$ m/s. It follows from these estimates that $c_i \gg a \approx c_w$. Hence, the integrand in Equation (22) can be simplified to:

$$F_{x,z}^i(k, ka, z) = \frac{1}{2} \mu_0 \Sigma_c \exp[-k(z_1 - z)]; \quad F_{x,z}^w = \frac{\exp(-kz)}{(c_w - ia)}. \quad (23)$$

Substituting Equation (23) into Equation (22), we arrive at the following equations for the components of the geomagnetic field perturbation:

$$\begin{aligned} \bar{b}_x^{(p,m)}(k, z) &= -B_0 \frac{\xi_{p,m}}{h} \left[\Phi_i^{(p,m)}(k, z) + \Phi_w^{(p,m)}(k, z) \right]; \\ \bar{b}_z^{(p,m)}(k, z) &= -iB_0 \frac{\xi_{p,m}}{h} \left[-\Phi_i^{(p,m)}(k, z) + \Phi_w^{(p,m)}(k, z) \right] \\ \Phi_i^{(p,m)}(k, z) &= \frac{ah\mu_0\Sigma_c \exp(z_1/2H)}{2(1-h/\gamma H)} G_{p,m}(k) (\Gamma + iq) \exp[-k(z_1 - z) - iqz_1] \\ \Phi_w^{(p,m)}(k, z) &= G_{p,m}(k) \frac{a \exp(-kz)}{c_w - ia} \end{aligned} \quad (24)$$

As seen from Equation (24), the tsunami-induced geomagnetic perturbation in the atmosphere results from two different sources. The first one is determined by the function $\Phi_i^{(p,m)}(k, z)$. This source is due to the ionospheric currents caused by the AGW propagating in the ionosphere. The second source contains the factor $\Phi_w^{(p,m)}(k, z)$ which describes the current due to seawater motion in tsunami wave.

Now we shall estimate the ratio between the sources of currents flowing in the ionosphere and in the seawater as it follows from Equation (24). For this purpose let us compare the function $\Phi_i^{(p,m)}(k, z)$ taken at the point $z = z_1$ with the function $\Phi_w^{(p,m)}(k, z)$ at the point $z = 0$. Assuming for the moment that $k \ll 1/\sqrt{hH}$, then this ratio can be reduced to:

$$\frac{|\Phi_i|}{|\Phi_w|} \sim \sqrt{\frac{h}{H}} \frac{\Sigma_c}{\Sigma_w} \exp(z_1/2H).$$

Setting $\Sigma_P = \Sigma_H = 10$ S, $\sigma_w = 4$ S/m, $h = 1$ km, $\Sigma_w = 4 \times 10^3$ S, $z_1 = 120$ km, $H = 8$ km and $\Sigma_c = 15$ S yields

$$\sqrt{\frac{h}{H}} \frac{\Sigma_c}{\Sigma_w} \exp(z_1/2H) \sim 3.$$

Consequently, the analysis of the geomagnetic field perturbations generated by the tsunami-driven motion of seawater and by the ionospheric plasma motion due to AGW propagation shows that the two effects are comparable in amplitude. It should be noted that the above estimates are based on the values of the height-integrated conductivities which are typical for the dayside ionosphere. The nighttime ionospheric conductivities are one order of magnitude smaller than the daytime ones. This means that the contribution of the ionospheric current to the geomagnetic perturbation decreases under nighttime conditions.

The results of numerical modelling of the geomagnetic perturbations in the atmosphere are shown in Figure 4 as function of the horizontal coordinate x measured on the seawater surface. In making the plot of \bar{b}_x and \bar{b}_z we have used Equation (22) and the above-mentioned values of the media parameters. Additionally we have assumed that the undisturbed magnetic field $B_0 = 5 \times 10^{-5}$ T, the sound speed in the atmosphere $c = 330$ m/s and the amplitude of tsunami wave is equal to 0.5 m. In this figure the upper (a) and lower (b) panels correspond to the sea depth $h = 1$ and 3 km, respectively.

It is obvious from Figure 4 that the amplitude of the geomagnetic perturbation on the sea surface increases with the sea depth h . It also appears that the seismic source type may greatly affect the x -dependence of the geomagnetic perturbation.

Figure 5 shows the x -dependence of the components of the geomagnetic perturbation in the ionosphere at the altitude 120 km. In making this plot we have used the same parameters. It is

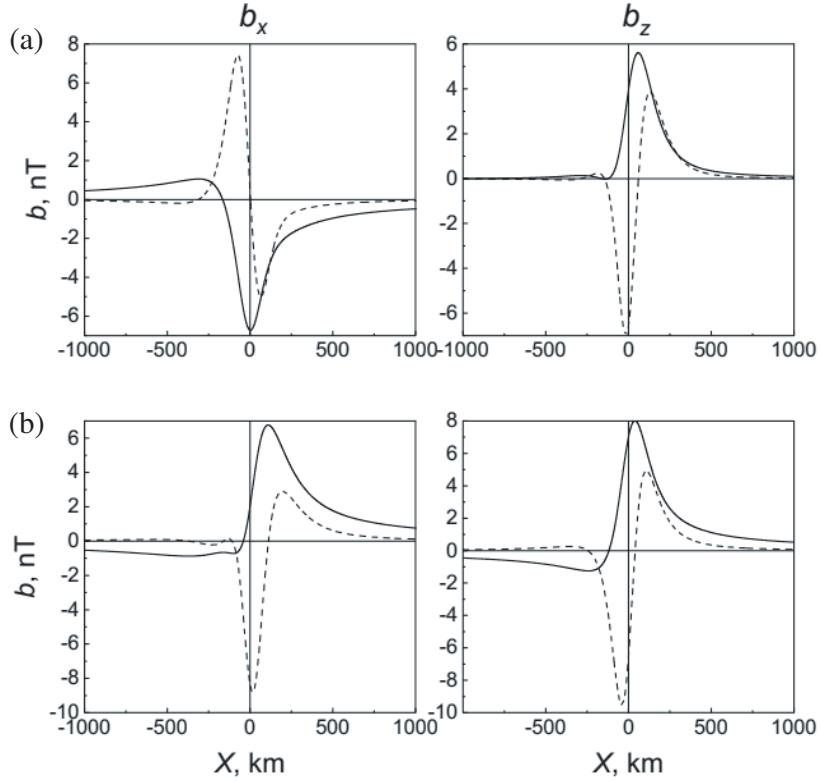


Figure 4. Model calculations of the amplitudes of the geomagnetic perturbation in the atmosphere versus horizontal coordinate x measured on the seawater surface. The perturbations caused by the “piston” and “membrane” earthquakes are shown with solid and dashed lines, respectively. The (a) upper and (b) lower panels correspond to $h = 1$ and 3 km, respectively.

evident from Figure 5 that the features of these plots essentially depend on the depth h of the sea medium. The horizontal component of the geomagnetic perturbation changes the sign with the increase in h whereas the vertical component varies from the single-polarity impulse to the bipolar one (solid lines) or from the bipolar impulse to three-polarity one.

5. PERTURBATION OF GEOMAGNETIC FIELD IN THE UPPER IONOSPHERE AND MAGNETOSPHERE

The geomagnetic perturbation in the upper ionosphere and magnetosphere builds up as a result of neutral gas motion in the tsunami-induced AGW followed by the generation of transverse and longitudinal currents in the ionosphere. In the altitude region $z > z_1$ the first equation of set (9) is reduced to

$$\mu_0 j_{\parallel}(x, t) = \frac{\partial b_y(x, t)}{\partial x}. \quad (25)$$

As we shall see, the inequality $E_y \ll E_x$ is valid for the ionosphere. Thus, one can neglect the component E_y in Equation (18) whence it follows that:

$$E_x(x, z, t) \approx \frac{\Sigma_H B_0}{2\Sigma_P} v_x(x, z_1, t); \quad j_{\parallel}(x, z, t) \approx \frac{1}{2}\Sigma_H B_0 \frac{\partial v_x(x, z_1, t)}{\partial x} \quad (26)$$

Rearranging Equations (25) and (26), we find the spatiotemporal distribution of the electromagnetic perturbations in the upper ionosphere and magnetosphere:

$$b_y(x, t) = \frac{1}{2}\mu_0 \Sigma_H B_0 v_x(x, z_1, t); \quad E_x(x, t) = \frac{1}{\mu_0 \Sigma_P} b_y(x, t). \quad (27)$$

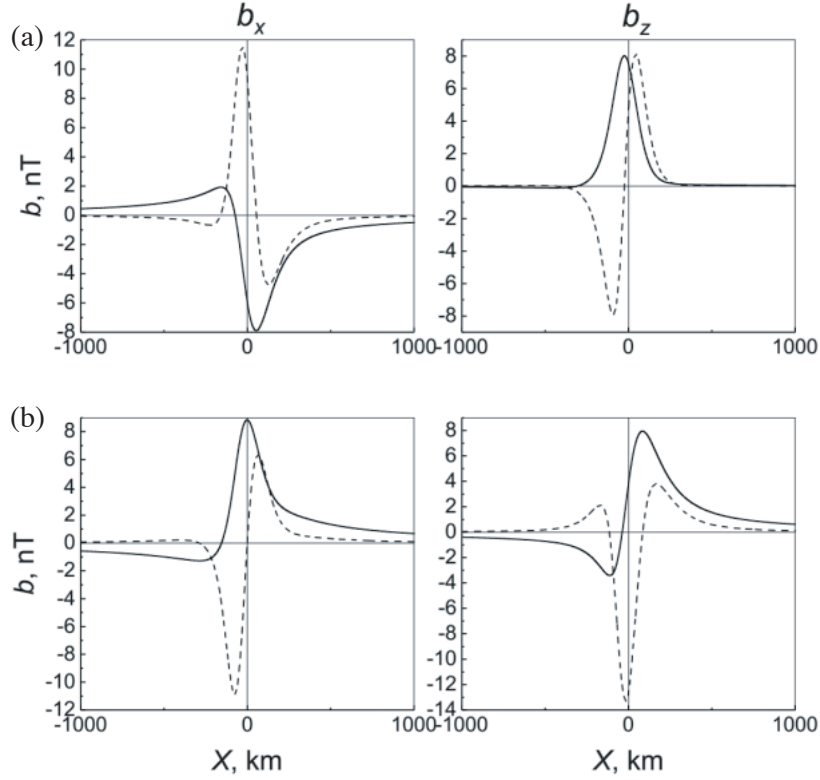


Figure 5. The same as in Figure 4 but for the geomagnetic perturbation in the ionosphere at the altitude 120 km.

It follows from Equation (27) that the amplitude of the electromagnetic perturbations in the region $z > z_1$ is proportional to the neutral gas velocity in the AGW. Substituting Equation (8) for v_x into Equation (27) yields

$$\begin{aligned}
 b_y^{(p,m)}(x,t) &= \frac{1}{\pi} \operatorname{Re} \int_0^{\infty} \bar{b}_y^{(p,m)}(k) \exp[ik(x-at)] dk; \\
 E_x(x,t) &= \frac{1}{\mu_0 \Sigma_P} b_y(x,t); \quad j_{\parallel}(x,t) = \frac{1}{\mu_0} \frac{\partial b_y(x,t)}{\partial x}; \\
 \bar{b}_y^{(p,m)}(k) &= -\frac{1}{2} B_0 \mu_0 \Sigma_H a \xi_{p,m} G_{p,m}(k) \frac{\Gamma + iq}{1 - h/\gamma H} \exp\left(\frac{z_1}{2H} - iqz_1\right).
 \end{aligned} \tag{28}$$

The results of numerical calculations of the geomagnetic perturbations and the field-aligned current in the upper ionosphere and magnetosphere are shown in Figure 6 as function of the horizontal coordinate x . In making the plot of b_y and j_{\parallel} we have used Equation (28) and the above-mentioned values of the media parameters. As is seen from Figure 6 the sea depth may greatly affect the shape of signals. The amplitude of the geomagnetic field variation reaches a peak value about 10 nT while the amplitude of the field-aligned current amounts to a value about $8 \times 10^{-8} \text{ A/m}^2$.

Now we focus our attention on the estimate of the electric field in the region $z > z_1$. It follows from equation $\nabla \times \mathbf{E} = -\partial \mathbf{b} / \partial t$ under condition $\partial / \partial t = -a \partial / \partial x$ that $E_y = ab_z$. It obvious from Figure 5 that $b_z \sim (1 \div 10) \text{ nT}$ at the altitudes of the ionosphere. Taking this value of b_z and assuming that $a = 100 \text{ m/s}$, yields $E_y \approx (0.1 \div 1.0) \mu\text{V/m}$. As is seen from Figure 6, the component $b_y \approx 10 \text{ nT}$. Substituting this value b_y and $\Sigma_P \approx (1 \div 10) \text{ S}$ into Equation (28) we get $E_x \approx (1 \div 10) \text{ mV/m}$. It follows from these estimates that $E_y \ll E_x$. The interpretation we make is that the component E_x of the geomagnetic perturbation in the ionosphere is present even in electrostatic approximation whereas

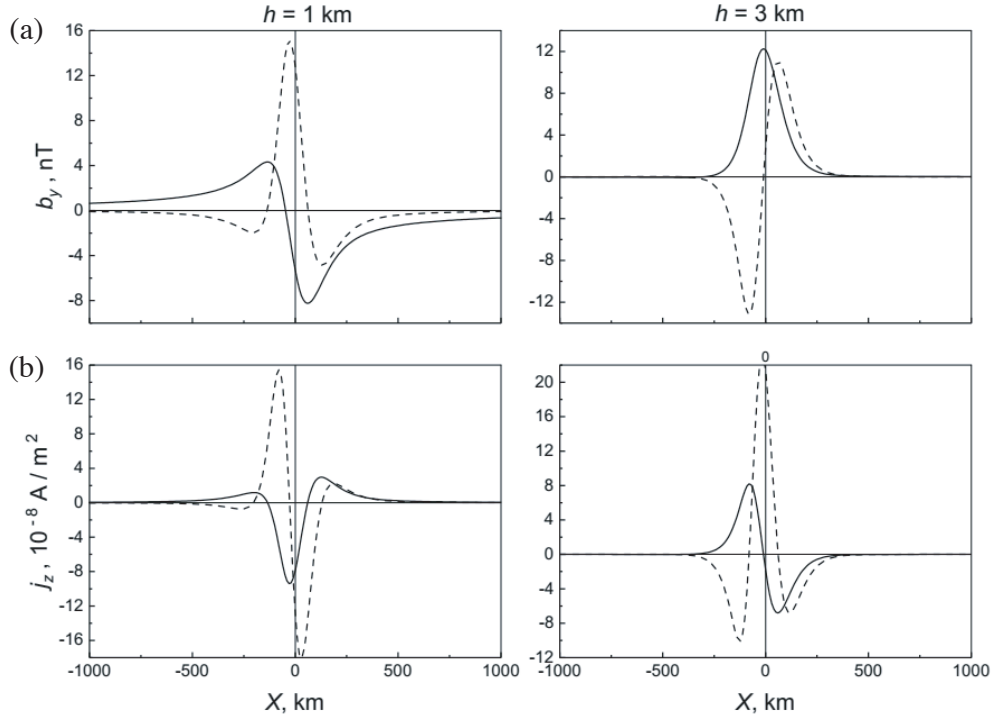


Figure 6. Model calculations of amplitude of the geomagnetic field component b_y (a) (upper panels) and the field-aligned current j_{\parallel} (b) (lower panels) in the upper ionosphere and magnetosphere as a function of the horizontal coordinate x . The perturbations caused by the “piston” and “membrane” earthquakes are shown with solid and dashed lines, respectively. The left and right panels correspond to $h = 1$ and 3 km, respectively.

the component E_y is generated due to only inductive effect; that is, E_y tends to zero in the electrostatic approximation.

The above estimates have shown that the tsunami-induced effect in the upper ionosphere and magnetosphere is more pronounced with the component E_x of the electric field and the component b_y of the geomagnetic perturbation caused by the field-aligned current. It follows from the above estimates that these components can be detectable at the satellite orbits.

6. DISCUSSION AND CONCLUSION

It is generally believed that the geomagnetic perturbation associated with tsunami wave propagation is mainly due to the generation of the electric currents caused by the tsunami-driven motion of conductive seawater. Our theoretical analysis has demonstrated that other physical mechanism associated with the formation of an electric current in the ionosphere can play a significant role in the generation of the geomagnetic perturbations. The ionospheric currents are caused by AGW propagating from the atmosphere into the ionosphere. The AGW originates from the vertical large scale displacement of the sea surface caused by the propagation of the tsunami wave, which in turn builds up as a result of seabed displacement due to subsea earthquake. Two different types of such earthquakes; that is, “piston” and “membrane” earthquakes were examined. The “piston” subsea earthquake is accompanied by the fast vertical displacement of sea floor followed by the formation of the residual sea floor displacement that results in the generation of a surface single-polarity gravity wave. The “membrane” subsea earthquake does not cause the residual sea floor displacement. This type of earthquakes is accompanied by the generation of the bipolar gravity wave.

Despite the ionospheric conductivity is much smaller than the seawater conductivity, the electric

current density produced by the AGW in the ionosphere can be greater than that generated by the tsunami wave in the seawater. The reason is that the amplitude of air velocity in AGW exponentially increases with altitude. The amplitude of the geomagnetic field perturbation has been shown to reach the value about 1–10 nT for the dayside parameters of the ionosphere. The contribution to the observed signal from the ionospheric currents decreases at nighttime conditions because the height-integrated conductivities of the nighttime ionosphere are one order of magnitude lower than that of the daytime ionosphere.

The electric current generated by AGW in the ionosphere is closed to the conjugate ionosphere through the field-aligned currents thereby exciting the transverse components of the geomagnetic and electric perturbations in the upper ionosphere and magnetosphere. The amplitude of the tsunami-produced geomagnetic perturbation and field-aligned current can amount to the values of 10 nT and 10^{-8} A/m², respectively, while the electric field reaches a peak value about 10 mV/m. It appears that such values of the electromagnetic perturbations can be detectable onboard the satellites. Thus, the satellite observation suggests broad potentials for application to monitor the tsunami wave from the space.

APPENDIX A.

In this section we focus our attention on the atmospheric AGW resulted from sea surface displacement. Let $\mathbf{v} = (v_x, 0, v_z)$ be the air velocity caused by the AGW propagation in the atmosphere. The projections of the air velocity onto coordinate axes and the perturbation of air pressure p is assumed to depend on variables x and t according to the harmonic law, that is

$$\begin{aligned} v_x(x, z, t) &= U(z) \exp(ikx - i\omega t), \\ v_z(x, z, t) &= W(z) \exp(ikx - i\omega t), \\ p(x, z, t) &= P(z) \exp(ikx - i\omega t), \end{aligned} \quad (\text{A1})$$

where U , W , and P are unknown functions of z . Since only real parts of these complex functions (A1) have a physical meaning, the flux density of the wave energy is given by $\mathbf{S} = \text{Re}(p)\text{Re}(\mathbf{v})$ [19]. Vertical component of \mathbf{S} averaged on x and t has to be positive because the wave energy extends from the source; that is, upward from the sea surface:

$$\langle S_z \rangle = \langle \text{Re}(p) \text{Re}(v_z) \rangle_{x,t} = \frac{1}{2} \text{Re}(PW^*) > 0, \quad (\text{A2})$$

where the symbol $*$ denotes a complex conjugate value.

Equation of motion for the pressure perturbation and the projection v_x is given by:

$$\rho_0 \frac{\partial v_x}{\partial t} = -\frac{\partial p}{\partial x}.$$

Substituting Equation (A1) for v_x and p into the above equation, yields

$$P = \frac{\rho_0 \omega}{k} U, \quad (\text{A3})$$

where $\rho_0(z) = \bar{\rho} \exp(-z/H)$ and $\bar{\rho}$ is the air density at the sea level. Substituting Equation (A3) for P into Equation (A2), we obtain the dependence of the wave energy flux density S_z on the gas velocity components:

$$S_z = \frac{\omega \bar{\rho}}{2k} \text{Re}(UW^*) \exp(-z/H) \quad (\text{A4})$$

Substituting Equation (A1) for v_x and v_z into Equation (A3) we come to set of equations for the functions $W(z)$ and $U(z)$. The general solution of this set is given by

$$\begin{aligned} W &= e^{z/2H} (Ae^{iqz} + Be^{-iqz}), \quad U = e^{z/2H} (\alpha Ae^{iqz} - \alpha^* Be^{-iqz}); \\ \alpha &= -ik(\Gamma - iq)/(k^2 - \omega^2/c^2), \end{aligned} \quad (\text{A5})$$

where Γ and q are the constants given by Equation (4) while A and B are undetermined coefficients. Substituting Equation (A5) for W and U into Equation (A4), we get:

$$S_z = \frac{\bar{\rho} \omega}{2k} \text{Re} \left[\alpha |A|^2 - \alpha^* |B|^2 \right] = -\frac{\bar{\rho} \omega q}{2(k^2 - \omega^2/c^2)} \left(|A|^2 - |B|^2 \right). \quad (\text{A6})$$

The AGW produced by tsunami propagates along x -axis at phase velocity $a = \omega/k$. Substituting this relationship into Equation (A6), we obtain

$$S_z = -\frac{\bar{\rho}\omega q}{2k^2(1 - a^2/c^2)} (|A|^2 - |B|^2). \quad (\text{A7})$$

Let $v_z(k, 0, \omega)$ be the amplitude of the air vertical velocity taken at the level $z = 0$. Thus, the boundary condition of the problem is $W(0) = v_z(k, 0, \omega)$. The other requirement is that the vertical component of the energy flux density has to be positive; that is, $S_z > 0$. These conditions makes it possible to find the constants A and B . Considering Equation (A7) for S_z and taking into account that $1 - \omega^2/(c^2k^2) = 1 - a^2/c^2 > 0$, the radiation condition $S_z > 0$ reduces to the following form

$$\begin{aligned} A = v_z(k, 0, \omega), \quad B = 0 & \quad \text{if } \text{sgn}(\omega) < 0; \\ A = 0, \quad B = v_z(k, 0, \omega) & \quad \text{if } \text{sgn}(\omega) > 0. \end{aligned} \quad (\text{A8})$$

Thus, the radiation condition depends on the sign of ω . Finally, substituting Equation (A8) for A and B into Equation (A5) and rearranging we come to the solution of the problem given by Equation (4).

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